Eliminating spatiotemporal chaos and spiral waves by weak spatial perturbations

Peng-Ye Wang,[1,2] and Ping Xie[1]
[1]Laboratory of Optical Physics, Institute of Physics and Center for Condensed Matter Physics, Chinese Academy of Sciences, P.O. Box 603, Beijing 100080, China
[2]CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China

(Received 7 September 1999)

The possibility of eliminating spatiotemporal chaos and spiral waves by weak spatial perturbations in a spatially extended dynamical system is demonstrated numerically through the example of a wide-aperture laser. The time-independent weak spatial perturbation can effectively migrate the system from the state of spatiotemporal chaos or spiral waves to that of traveling waves. The threshold and the controllable range of the control parameters are given. By varying the amplitude or the spatial wave vector of the perturbation, drastic changes in the spatiotemporal dynamics are found.

PACS number(s): 05.45.−a, 42.65.Sf, 47.54.+r

Spatiotemporal chaos and spiral waves exist extensively in dynamical systems with spatial extension, such as cardiac tissue, the Belousov-Zhabotinsky reaction, hydrodynamical systems, and optical systems. In some cases spatiotemporal chaos and spiral waves are undesirable because of their harmfulness. For example, a strong tornado can do great damage to human beings; fibrillation in the ventricular myocardium causes fatal cardiac diseases. Therefore, an effective way of eliminating them is highly desirable. Since the first suggestion of controlling chaos by Ott, Grebogi, and Yorke (OGY) [1], considerable research has been focused on this field of controlling both temporal [1–14] and spatiotemporal chaos [15–20]. The method developed to control temporal chaos can be classified into three main schemes: discrete feedback [1–5], continuous feedback [6,7], and weak periodic perturbation [8–14]. Early efforts toward suppression of spatiotemporal chaos were made by the extension of the OGY algorithm [15,16]. Later Lu *et al.* [19] extended the delayed feedback method suggested by Pyragas [6] to control spatiotemporal chaos in an optical system. Martin *et al.* [20] suggested a Fourier space method to control unstable patterns in a mean-field model for a two-level medium in an optical cavity. Hochheiser *et al.* [21] applied a spatial filter with delayed feedback to a wide-aperture laser model for controlling optical turbulence. Phase Fourier filters were also used in manipulating spatiotemporal dynamics in a spatially extended nonlinear optical system with a feedback [22]. Mamaev and Saffman [23] and Jensen *et al.* [24] experimentally suppressed optical turbulence by Fourier plane filtering. On the other hand, controlling spiral waves has also attracted much attention recently [25] due to its great potential applications. Herein, we wish to find out whether a time-independent weak spatial perturbation, which has been successfully used to stabilize, select, and track unstable spatial patterns both theoretically [26] and experimentally [27], can eliminate spatiotemporal chaos and spiral waves in a spatially extended dynamical system. In this work the spatial perturbation method is used to control spatiotemporal dynamics which do not exist in the system of Ref. [26] where all the patterns are time independent. Although our method is intrusive because more or less variations are introduced compared to the original system and the perturbation does not disappear, it is a powerful way of controlling spatiotemporal dynamics from the point of view of practical usefulness due to its experimental feasibility.

For a demonstration of our method, we use a coherently optically pumped two-level laser model which has received considerable attention in the investigation of chaotic dynamics [28]. The spatially extended laser system is described by the normalized Maxwell-Bloch equations with transverse coupling through diffraction:

$$\frac{\partial E}{\partial t}=\sigma(i\delta-1)E+\sigma P+i\nabla_\perp^2 E, \tag{1}$$

$$\frac{\partial P}{\partial t}=-(i\delta+1)P+RE-ED, \tag{2}$$

$$\frac{\partial D}{\partial t}=-bD+\tfrac{1}{2}(EP^*+E^*P), \tag{3}$$

where E, P, and D are the electric field, atomic polarization, and population inversion, respectively, $\nabla_\perp^2$ is the two-dimensional transverse Laplacian, σ and b are the damping rate of the electric field in the cavity and the damping rate of the population inversion, respectively, normalized to the damping rate of the atomic polarization, δ is the detuning between the electric field and the resonator, and R is the pump parameter. Considering experimental feasibility, we apply a spatial perturbation to the pump. The perturbed pump can be written as

$$R=R_0[1+\alpha f(\mathbf{r})], \tag{4}$$

where R_0 is the unperturbed pump, α is the amplitude of the perturbation, and $f(\mathbf{r})$ is the two-dimensional spatial perturbation function with an amplitude of unity. In order to keep the perturbation weak, the perturbation amplitude α should be much smaller than 1. In cases without spatial perturbation, previous theoretical and experimental studies [29] in lasers and nonlinear optical systems have shown that these spatially extended optical systems ubiquitously possess spatiotemporal chaos and spiral waves. We wish our perturbation function in Eq. (4) to have a taming effect on spatiotemporal

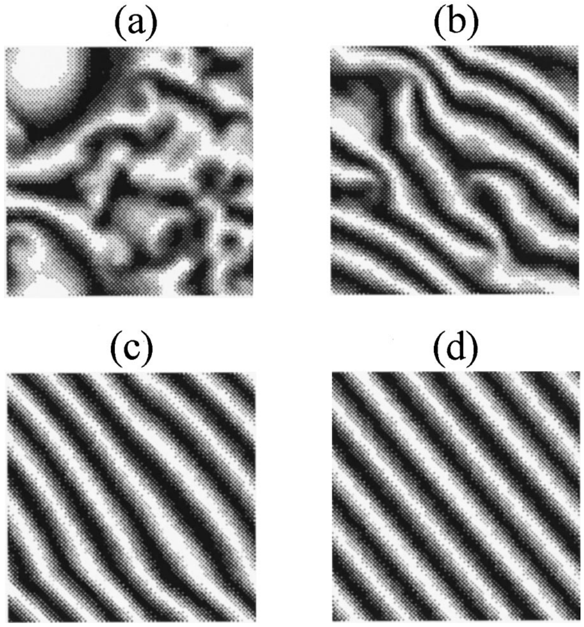

FIG. 1. Temporal evolution of the transverse field distribution [Re(E)] for the process of eliminating spatiotemporal chaos. The time after exerting the spatial perturbation is (a) $t=0$ (no perturbation), (b) $t=6000$, (c) $t=19\,700$, and (d) $t=26\,000$. The parameters are $\delta=0.25$, $R_0=1.8$, and $k=1.152$. In all figures the parameters and variables are dimensionless unless otherwise indicated.

chaos and spiral waves and to migrate the system to the desired uniform traveling wave state. Recalling the approaches to controlling temporal chaos by using a weak periodic forcing [8–14] and the form of spatial perturbation in stabilizing roll patterns [26], we choose the perturbation function as $f(\mathbf{r})=e^{i\mathbf{k}\cdot\mathbf{r}}$, which is the spatial part of a traveling wave, where $\mathbf{k}$ is the wave vector with a magnitude of $k=|\mathbf{k}|$.

The results that we will present were obtained by numerically integrating Eqs. (1)–(3) with the spatially perturbed pump defined in Eq. (4). The integrations were performed using a split step spectral method on a 64×64 grid with a box size of 16π. We fix the parameters $\sigma=3$ and $b=1$. A typical result of eliminating spatiotemporal chaos is shown in Fig. 1, where the real part of the field is presented in gray scale. Figure 1(a) is the spatiotemporal chaotic state before the spatial perturbation is exerted on the system. From the successive Figs. 1(b), 1(c), and 1(d) obtained after the spatial perturbation is exerted, we can see that the spatiotemporal chaos is effectively eliminated and a uniform traveling wave is finally obtained. The amplitude of the perturbation used is $\alpha=0.045$. In this case, the threshold value for the amplitude is around $\alpha_{\mathrm{th}}\approx0.032$. If the value of α is smaller than α_{th} the spatiotemporal chaos cannot be eliminated. The spatial power spectra of the field corresponding to Fig. 1 are shown in Fig. 2, from which we can clearly see that all the chaotic components disappear gradually after the spatial perturbation is exerted and finally only the component of the traveling wave can be found. Figure 3 shows the local temporal wave forms of the real part and the intensity of the field at the center of the integration box. As expected, the real part and the intensity of the field become periodic and constant, respectively, after the system has migrated to the traveling

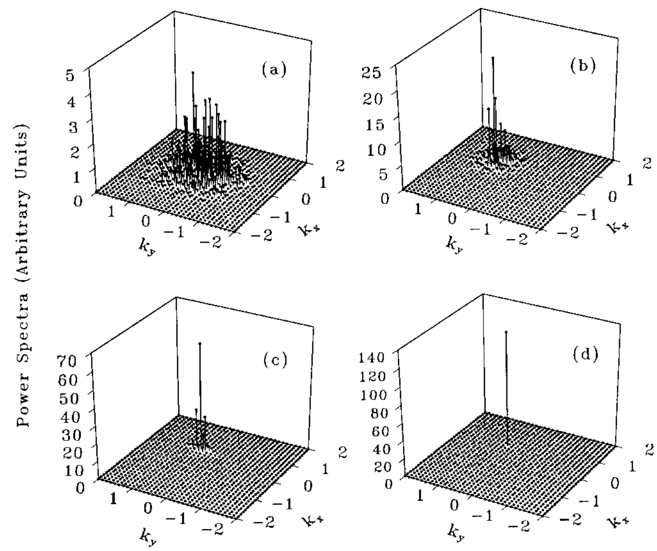

FIG. 2. Spatial power spectra of the field (E) corresponding to Fig. 1.

wave state because of the spatial perturbation. A typical result of eliminating spiral waves is shown in Fig. 4, with the corresponding spatial power spectra shown in Fig. 5. Figures 4(a) and 5(a) show the original spiral wave without spatial perturbation. We can see that the rotation of the spiral wave is first broken by the spatial perturbation and a pacemaker appears at the position of each spiral core at an early stage of the elimination [Fig. 4(b)]. Gradually, only the local defect at the position of the original spiral core that is most robust exists [Fig. 4(c)]. Finally, the system asymptotically evolves to the final traveling wave state [Fig. 4(b)] with a single spatial Fourier component [Fig. 5(d)]. The amplitude of the spatial perturbation used is $\alpha=0.045$. In this case the threshold value of the amplitude of the spatial perturbation for eliminating the spiral waves is around $\alpha_{\mathrm{th}}\approx0.035$, and α must be larger than α_{th} in order to eliminate the spiral waves.

In order to explore the global responses of the system to the spatial perturbation, we systematically studied, in detail, the spatiotemporal dynamics in the plane of the control pa-

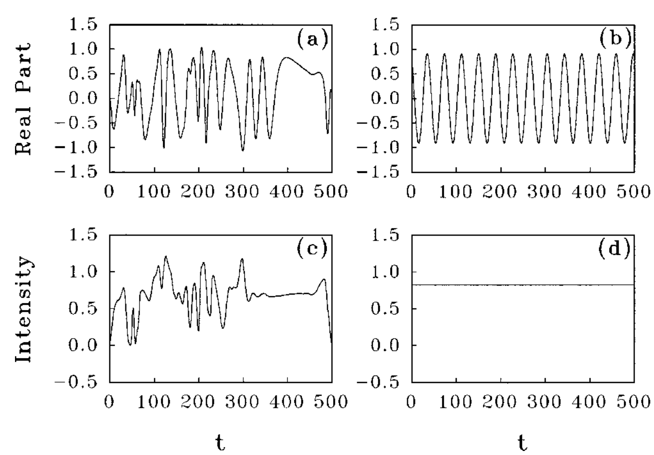

FIG. 3. Wave forms of the local field at the center of the integrating box: (a) Real part without spatial perturbation (spatiotemporal chaos), (b) real part with spatial perturbation (traveling wave), (c) intensity corresponding to (a) and (d) intensity corresponding to (b). The parameters are the same as in Fig. 1.

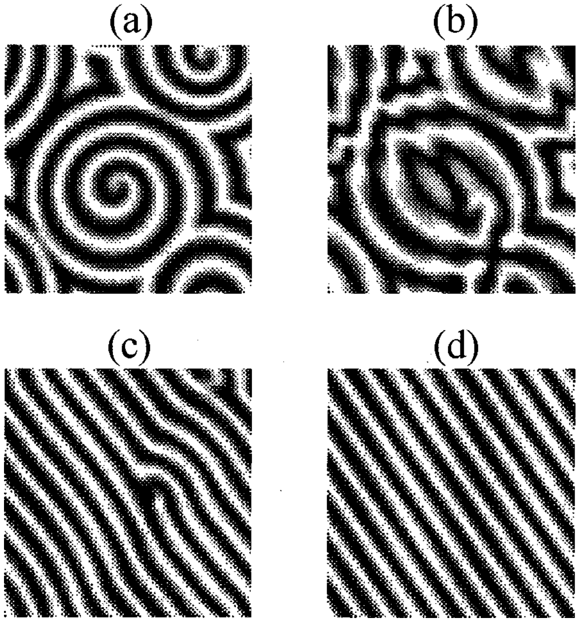

FIG. 4. Temporal evolution of the transverse field distribution [Re(E)] for the process of eliminating spiral waves. The time after exerting the spatial perturbation is (a) $t=0$ (no perturbation), (b) $t=40$, (c) $t=2000$, and (d) $t=6000$. The parameters are $\delta=0$, $R_0=5$, and $k=1.152$.

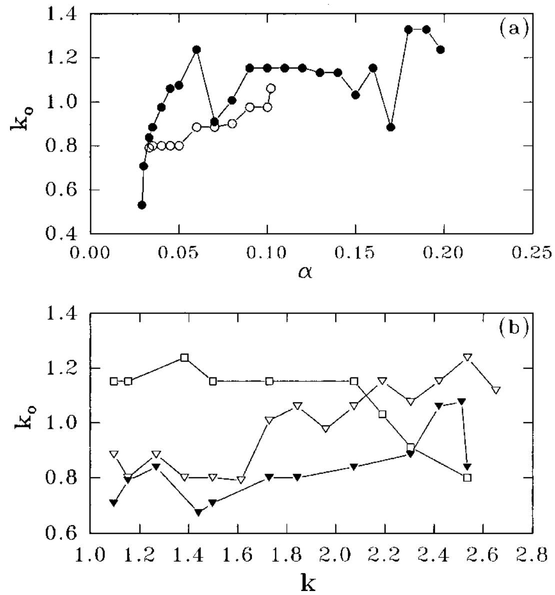

FIG. 6. Magnitude k_o of the output traveling wave vector versus (a) the amplitude α of the perturbation and (b) the magnitude k of the wave vector of the perturbation. The points represent actual results and they are connected by lines to guide the eye. The parameter values: solid dots, $k=2.074$; circles, $k=1.152$; squares, $\alpha=0.12$; empty triangles, $\alpha=0.045$; solid triangles, $\alpha=0.033$.

rameters (the amplitude α and the magnitude k of the wave vector of the perturbation signal). We find that in a large range of α and k the spatiotemporal chaos and spiral waves can be effectively eliminated and the system migrates to a traveling wave state (we say it is controllable). For each k value there exists a threshold α_{th} of the perturbation amplitude α. Only when $\alpha > \alpha_{\mathrm{th}}$ is the system controllable. Below α_{th} the traveling waves become unstable (the Eckhaus instability). The threshold α_{th} is dependent on the value of k, but stays small in the whole controllable range of k. For example, for $\delta=0.25$ and $R_0=1.8$, we get $\alpha_{\mathrm{th}}\approx 0.02$ for $k=1.729$ and $\alpha_{\mathrm{th}}\approx 0.03$ for $k=2.074$. For $\alpha=0.033$, the controllable range for k is between 1.095 and 2.535. We thus see that even in the case of a very small perturbation amplitude the control is successful in a large range of k. For large α values, we find that there exists an upper limit α_{UL} of α for each k. For example, we get $\alpha_{\mathrm{UL}}\approx 0.198$ for $k=2.074$. Defect-mediated turbulence appears above this limit. It is interesting to show the dependence of the magnitude k_o of the wave vectors of the controlled traveling waves on the control parameters α and k in the controllable region. Some results are shown in Fig. 6. It is seen that they are nonmonotonic and the value of k_o is not the same as that of k. These results demonstrate how a time-independent spatial perturbation, even if relatively weak, can change the dynamics of the system in a drastic fashion—spatiotemporal chaos and spiral waves can be eliminated, stable traveling waves can be obtained, and nonmonotonic behavior of k_o versus k and α emerges. Similar phenomena were found in the temporal stabilization of multimode solid state lasers by using small periodic modulations [14]. The mechanism can be understood as follows. Because the spatiotemporal dynamics is extremely sensitive to variations of the system parameters, a very weak perturbation to one of the system parameters can effectively migrate the spatiotemporal chaotic state to a periodic one. On the other hand, if the perturbation is too strong (e.g., the amplitude of the perturbation signal is comparable to that of the laser output) the external force will compete with the intrinsic dynamics. This competition induces new instabilities.

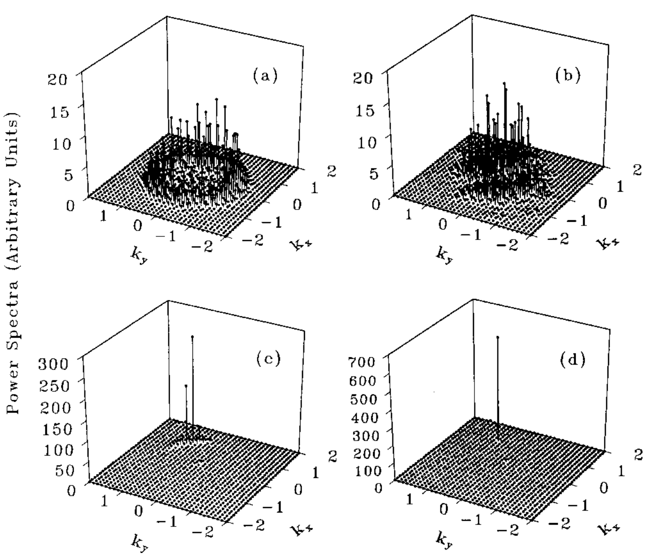

FIG. 5. Spatial power spectra of the field (E) corresponding to Fig. 4.

In conclusion, we have shown that a weak spatial perturbation can effectively eliminate spatiotemporal chaos and spiral waves. With the example of a wide-aperture laser system, we have demonstrated that a time-independent spatial perturbation migrates a spatially extended dynamical system from a state of spatiotemporal chaos or spiral waves to a uniform traveling wave state. These results may present a clue to understanding why tornadoes are usually and more easily born in plains (because fewer spatial perturbations exist there) and suggest construction of proper obstacles on the ground, as weak spatial perturbations, to eliminate them. Due to the experimental feasibility of realizing spatial perturbations, this method may also be used to eliminate fatal cardiac fibrillation, which is widely believed to be the breakup of a spiral wave of electrical activity into multiple spirals leading to a spatiotemporal chaotic state [30]. Therefore, our spatial perturbation method of controlling spatiotemporal dynamics is of general relevance and great importance.

This research was supported by the National Natural Science Foundation of China.

[1] E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).
[2] W. L. Ditto, S. N. Rauseo, and M. L. Spano, Phys. Rev. Lett. **65**, 3211 (1990); U. Dressler and G. Nitsche, *ibid.* **68**, 1 (1992).
[3] B. Peng, V. Petrov, and K. Showalter, J. Phys. Chem. **95**, 4957 (1991).
[4] E. R. Hunt, Phys. Rev. Lett. **67**, 1953 (1991); R. Roy, T. W. Murphy, T. D. Maier, Z. Gills, and E. R. Hunt, *ibid.* **68**, 1259 (1992).
[5] P. Parmananda, P. Sherard, R. W. Rollins, and H. D. Dewald, Phys. Rev. E **47**, R3003 (1993).
[6] K. Pyragas, Phys. Lett. A **170**, 421 (1992); K. Pyragas and A. Tamasevicius, *ibid.* **180**, 99 (1993).
[7] W. Just, T. Bernard, M. Ostheimer, E. Reibold, and H. Benner, Phys. Rev. Lett. **78**, 203 (1997).
[8] R. Lima and M. Pettini, Phys. Rev. A **41**, 726 (1990).
[9] Y. Braiman and I. Goldhirsch, Phys. Rev. Lett. **66**, 2545 (1991).
[10] A. Azevedo and S. M. Rezende, Phys. Rev. Lett. **66**, 1342 (1991).
[11] L. Fronzoni, M. Giocondo, and M. Pettini, Phys. Rev. A **43**, 6483 (1991).
[12] R. Chacón and J. Díaz Bejarano, Phys. Rev. Lett. **71**, 3103 (1993).
[13] R. Meucci, W. Gadomski, M. Ciofini, and F. T. Arecchi, Phys. Rev. E **49**, R2528 (1994).
[14] P. Colet and Y. Braiman, Phys. Rev. E **53**, 200 (1996).
[15] J. A. Sepulchre and A. Babloyantz, Phys. Rev. E **48**, 945 (1993).
[16] G. Hu and Z. L. Qu, Phys. Rev. Lett. **72**, 68 (1994); D. Auerbach, *ibid.* **72**, 1184 (1994).
[17] F. Qin, E. E. Wolf, and H. C. Chang, Phys. Rev. Lett. **72**, 1459 (1994); I. Aranson, H. Levine, and L. Tsimring, *ibid.* **72**, 2561 (1994); V. Petrov, S. Metens, P. Borckmans, G. Dewel, and K. Showalter, *ibid.* **75**, 2895 (1995); A. Hagberg, E. Meron, I. Rubinstein, and B. Zaltzman, *ibid.* **76**, 427 (1996).
[18] G. A. Johnson, M. Löcher, and E. R. Hunt, Phys. Rev. E **51**, R1625 (1995); C. Lourenço, M. Hougardy, and A. Babloyantz, *ibid.* **52**, 1525 (1995).
[19] W. Lu, D. Yu, and R. G. Harrison, Phys. Rev. Lett. **76**, 3316 (1996); **78**, 4375 (1997).
[20] R. Martin, A. J. Scroggie, G.-L. Oppo, and W. J. Firth, Phys. Rev. Lett. **77**, 4007 (1996).
[21] D. Hochheiser, J. V. Moloney, and J. Lega, Phys. Rev. A **55**, R4011 (1997).
[22] M. A. Vorontsov and B. A. Samson, Phys. Rev. A **57**, 3040 (1998).
[23] A. V. Mamaev and M. Saffman, Phys. Rev. Lett. **80**, 3499 (1998).
[24] S. J. Jensen, M. Schwab, and C. Denz, Phys. Rev. Lett. **81**, 1614 (1998).
[25] O. Steinbock, V. Zykov, and S. C. Müller, Nature (London) **366**, 322 (1993); M. Braune and H. Engel, Chem. Phys. Lett. **211**, 534 (1993); V. Petrov, Q. Ouyang, and H. L. Swinney, Nature (London) **388**, 655 (1997); S. Grill, V. S. Zykov, and S. C. Müller, Phys. Rev. Lett. **75**, 3368 (1995); V. S. Zykov, A. S. Mikhailov, and S. C. Müller, *ibid.* **78**, 3398 (1997); G. Hu, J. H. Xiao, L. O. Chua, and L. Pivka, *ibid.* **80**, 1884 (1998).
[26] Peng-Ye Wang, Ping Xie, Jian-Hua Dai, and Hong-Jun Zhang, Phys. Rev. Lett. **80**, 4669 (1998).
[27] M. Ciofini, A. Labate, R. Meucci, and Peng-Ye Wang, Opt. Commun. **154**, 307 (1998); R. Meucci, A. Labate, M. Ciofini, and Peng-Ye Wang, Quantum Semiclassic. Opt. **10**, 803 (1998); Peng-Ye Wang and M. Saffman, Opt. Lett. **24**, 1118 (1999).
[28] C. O. Weiss and R. Vilaseca, *Dynamics of Lasers* (VCH, Weinheim, 1991); *Nonlinear Dynamics and Spatial Complexity in Optical Systems*, edited by R. G. Harrison, J. S. Uppal, and P. Osborne (SUSSP/Institute of Physics, Bristol, 1993).
[29] L. A. Lugiato, C. Oldano, and L. M. Narducci, J. Opt. Soc. Am. B **5**, 879 (1988); P. Coullet, L. Gil, and F. Rocca, Opt. Commun. **73**, 403 (1989); F. T. Arecchi, G. Giacomelli, P. L. Ramazza, and S. Residori, Phys. Rev. Lett. **67**, 3749 (1991); P. K. Jakobsen, J. V. Moloney, N. C. Newell, and R. Indik, Phys. Rev. A **45**, 8129 (1992); D. Yu, W. Lu, and R. G. Harrison, Phys. Rev. Lett. **77**, 5051 (1996).
[30] A. Karma, Phys. Rev. Lett. **71**, 1103 (1993); J. M. Davidenko, A. V. Pertsov, R. Salomonsz, W. Baxter, and J. Jalife, Nature (London) **355**, 349 (1992); R. A. Gray, A. M. Pertsov, and J. Jalife, *ibid.* **392**, 75 (1998); F. X. Witkowski, L. J. Leon, P. A. Penkoske, W. R. Giles, M. L. Spano, W. L. Ditto, and A. T. Winfree, *ibid.* **392**, 78 (1998).